

# Signature of Randall-Sundrum Quantum Gravity model in $\gamma\gamma$ scattering in the TeV range.

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## Abstract

We examine the implications of the Randall-Sundrum gravity models on  $\gamma\gamma$  scattering in the TeV range.

It has recently been proposed that the scale of weak interactions  $M \sim 1TeV$  is the only fundamental scale of energy in the description of particle interactions. The conventional scale of gravitational interactions, the Planck scale  $M_{pl} \gg M$  is no longer fundamental but arises out of a Kaluza-Klein compactification of a higher dimensional space-time into four dimensions. In the first such proposals of weak scale quantum gravity (WSQG), one starts with  $(4+n)$ -dimensional space-time with the standard model (SM) fields confined to a three-brane that we live in [1]. Gravity, on the other hand, is postulated to propagate in the full space-time including the  $n$  extra dimensions which are assumed to be compact of size  $r_c$ . For distances  $r \ll r_c$ , gravity is no different from other SM interactions, but for  $r \gg r_c$  the net effect of propagation of gravitons in the extra compact dimensions is to make strength of the gravitational interactions from  $1/M$ , via the relation:

$$M_{pl}^2 \sim M^{n+2} \cdot r_c^2. \quad (1)$$

For  $M \sim 1 - 10TeV$ , this last equation gives as low as  $1mm$ , which is the current experimental limit of validity of Newtonian gravity law. This scenario has interesting phenomenological implications in TeV-scale physics [2] as well as in the precise values of certain experimentally measurable low energy parameters like  $(g-2)$  of the muon [3]. The existence of the  $n$ -compact dimensions of course

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implies that the graviton is accompanied by a tower of massive cousins. These will be individually gravitationally coupled but added up, these effectively give rise to interactions comparable or even greater than in strength of weak interactions at sufficiently high energies in the TeV scale. Amongst the TeV-scale processes, clearly the ones that will be most sensitive to these ‘new’ interactions are the ones which are sufficiently suppressed in the SM. The scattering process  $\gamma\gamma \rightarrow \gamma\gamma$  is one such example since in the SM, the amplitude for this process is of  $O(\alpha^2)$  [4]. This is also a process in which there is a possibility of obtaining TeV-scale data in the near future and thus is worth investigating. Such investigations in the context of the ADD-model just outlined have been done in [5] and [6].

An alternative scenario of compactification has recently been proposed by Randall and Sundrum [7] where, the hierarchy of scales  $M_{pl}$  and  $M$  are generated in a different way. The space-time is now a 5D-non factorizable geometry, with a compact angular co-ordinate  $\phi$  ranging from  $|\phi| = 0$  to  $\pi$ . Two 3-branes with opposite tensions reside at the points  $\phi = 0$  and  $\phi = \pi$  and the resultant 4-d metric has the form

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} \cdot dx^\mu dx^\nu + r_c^2 d\phi^2, \quad (2)$$

with  $\mu, \nu = 0, 1, 2, 3$  and  $\sigma(\phi) = kr_c|\phi|$ ,  $r_c$  being the compactification scale. Our universe in this scenario is at  $\phi = \pi$  and because of the exponential ‘warp’ factor in the metric, physical masses in our 4-d world are related to the fundamental scale  $M_0$  of the theory by the relation  $M_{phys} = \exp(-kr_c\pi)M_0$ . This then is the reason for the weak scale arising with a value of  $kr_c \sim 10$ . Further, because of this compact dimension, gravitons will occur as a tower of particles. The zero mode is uniform in  $\phi$  and behaves like an ordinary graviton with a 4-d  $M_{pl}$  scale; the excitations are massive and couple with matter with a scale  $\Lambda = \exp(-kr_c)M_{pl}$  which is of the order of  $1TeV$ . The net Lagrangian of the coupling of this tower of gravitons with matter can then be expressed as a Lagrangian [8]:

$$L = - \left( \frac{1}{M_{pl}} \right) T^{\alpha\beta}(x) h_{\alpha\beta}^{(0)}(x) - \left( \frac{1}{\Lambda} \right) T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h_{\alpha\beta}^{(n)}(x). \quad (3)$$

The scenario in the Randall-Sundrum (RS) version of weak scale quantum gravity is thus substantially different from the ADD one, where the excited gravitons effectively form a continuum. In contrast the RS-excitations form a discrete set with well defined and calculable spectrum. We of course have no hint where exactly will the RS-excitations lie in the TeV-scale. However, choosing the mass of the lowest excitation as a parameter, it is possible to calculate the spectrum and this is what was done in the first paper on the phenomenology of the RS-model [9]. We will also consider the mass of the first RS-resonance as a parameter and investigate the behaviour of  $\gamma\gamma$ -scattering amplitude in this scenario. This is thus an extension of the investigations in references by Davoudiasl and by us, where changes in  $\gamma\gamma$  cross sections arising from weak scale quantum gravity

were investigated in the ADD scenario [5][6]. Should there be indications of departure from the SM in experiments in the near future, the difference between the two available pictures would be useful. This present note addresses thus the question of RS-scenario implication for the  $\gamma\gamma$  scattering.

We shall assume, that as in the first paper on RS-phenomenology, that the first RS-resonance occurs at a value at 600 GeV; changes in that would only shift the graphs but not the behaviour. Further, arguments based on string theory tells us that the parameter  $k/M_{pl} \sim 10^{-2}$ . We will therefore investigate the phenomenology in a range of  $k/M_{pl}$  around that.

We record first the standard model (SM) amplitudes for  $\gamma\gamma$  scattering. This is dominated by the W-loop and we take a region of phase space such that  $s$ ,  $|t|$  and  $|u|$  are much greater than  $M_w^2$  (that would limit the scattering angle in the c.m. to be not too close to either 0 or  $\pi$ , which of course is experimentally very convenient). Under these approximations, the SM expressions for the helicity amplitudes are, in terms of the standard Mandelstam variables  $s$ ,  $t$ ,  $u$ :

$$M_{++++}(s, t, u) = (-i \cdot 16\pi\alpha^2) \cdot \left( \log \left| \frac{u}{M_w^2} \right| \cdot \frac{s}{u} + \log \left| \frac{t}{M_w^2} \right| \cdot \frac{s}{t} \right) \quad (4)$$

$$\begin{aligned} M_{+-+}(s, t, u) = & \left( -i \cdot 12\pi\alpha^2 \frac{s-t}{u} \right) + \left( i \cdot \frac{8\pi\alpha^2}{u^2} \right) + (4u^2 - 3s) \left( \log \left| \frac{t}{u} \right| \right) \\ & - (i \cdot 16\pi\alpha^2) \left( \frac{u}{s} \log \left| \frac{u}{M_w^2} \right| + \frac{u^2}{st} \log \left| \frac{t}{M_w^2} \right| \right) \end{aligned} \quad (5)$$

$$M_{+--+}(s, t, u) = M_{+-+}(s, t, u) \quad (6)$$

All other amplitudes, not related to these are negligible in the region considered.

The SM-amplitudes have to be added to the contributions arising out of the exchange of RS-resonances in the  $s$ -,  $t$ - and  $u$ -channels. If we assume that the first of the resonances is in the sub-TeV range (assumed  $\sim 600\text{GeV}$  in our calculation), then up to about  $\sqrt{s} = 1\text{TeV}$ , it is a good approximation to consider the exchange of the lowest RS-resonance only and we will do that. Using the interaction given in equation (3), the RS-resonance contributions to the amplitudes work out to be:

$$M_{++++}^{RS} = \left( -\frac{s^2}{\Lambda^2} \right) [D(t) + D(u)] \quad (7)$$

$$M_{+-+}^{RS} = \left( -\frac{u^2}{\Lambda^2} \right) [D(s) + D(t)] \quad (8)$$

$$M_{+--+}^{RS} = \left( -\frac{t^2}{\Lambda^2} \right) [D(s) + D(u)] \quad (9)$$

where  $D(t) = (t - m_1^2)^{-1}$ ,  $D(u) = (u - m_1^2)^{-1}$  and  $D(s) = (s - m_1^2 + im_1 \cdot \Gamma + \frac{1}{4}\Gamma^2)^{-1}$ .

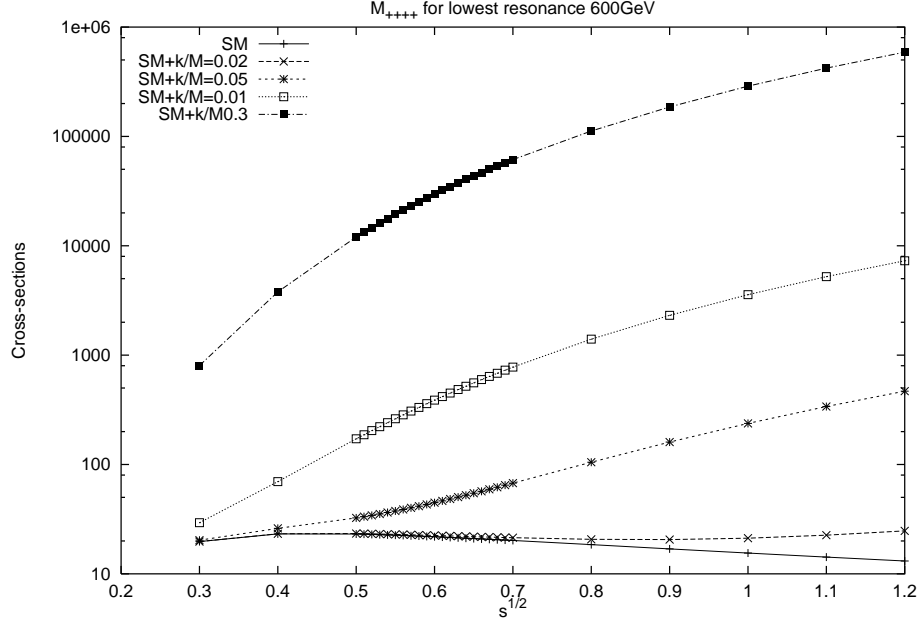


Figure 1: Magnitudes of the SM cross-sections and SM plus Randall-Sundrum contributions for initial helicities ++ for a lowest resonance of 600 GeV.

$m_1$  is the mass of the first RS-resonance and  $\Gamma$  its width, which can be calculated as:

$$\Gamma = \left( \frac{3}{10\pi} \right) (m_1 \cdot x_1^2) \left( \frac{k}{M_{pl}} \right)^2 \quad (10)$$

$x_1$  above is the first zero of the Bessel function  $J_1(x)$ .

The total amplitude  $M^{SM} + M^{RS}$  is therefore determined in terms of the parameter  $(k/M_{pl})$  for a given value of  $m_1$ .

Figures 1,2 shows the typical way in which the  $\gamma\gamma$  cross-sections for parallel and antiparallel initial helicity states behave as a function of the c.m. energy. Also shown are the pure SM-predictions for comparison. To see the nature of the difference between the RS- and ADD-scenario, we have also presented in figures 3 and 4, similar cross-sections for the ADD-scenario with a cut-off value of  $M_s = 3TeV$ . We have chosen for the calculations values of  $\sqrt{s}$  up to  $1TeV$ , hopefully because such energies would be accessible at NLC for  $\gamma\gamma$  processes using scattering of laser photons from  $e^+e^-$  beams.

The most striking feature of the RS-scenario, is the huge enhancement of the cross-sections even for low values of the parameter  $k/M_{pl}$  that we have considered. In comparison for the pure fermionic process  $e^+e^- \rightarrow \mu^+\mu^-$  considered by Davoudiasl, Hewett and Rizzo [9], the cross section rise is not so spectacular even for a higher range of values of the parameter  $k/M_{pl}$ . A second feature worth

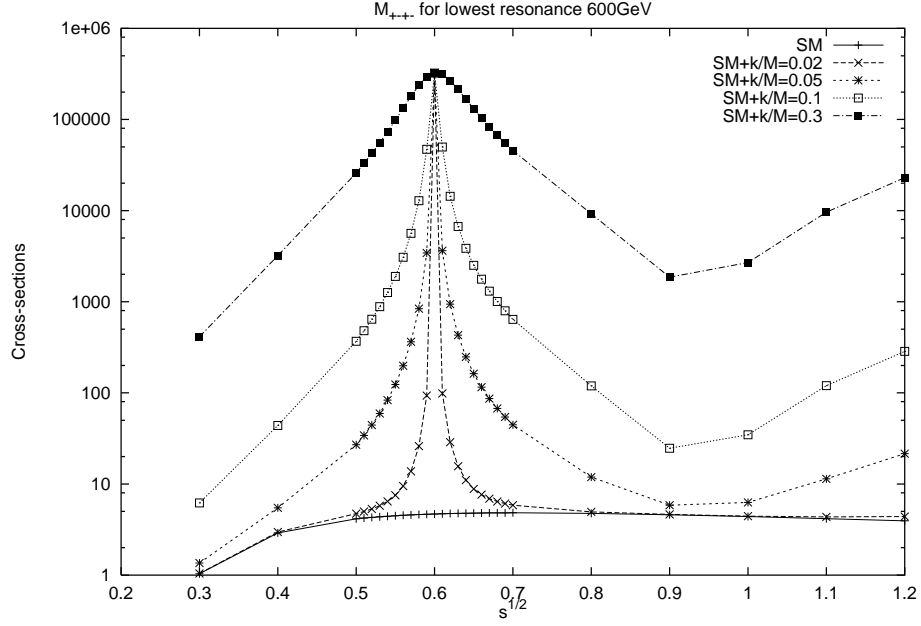


Figure 2: Magnitudes of the SM cross-sections and SM plus Randall-Sundrum contributions for initial helicities  $\pm -$  for a lowest resonance of 600 GeV.

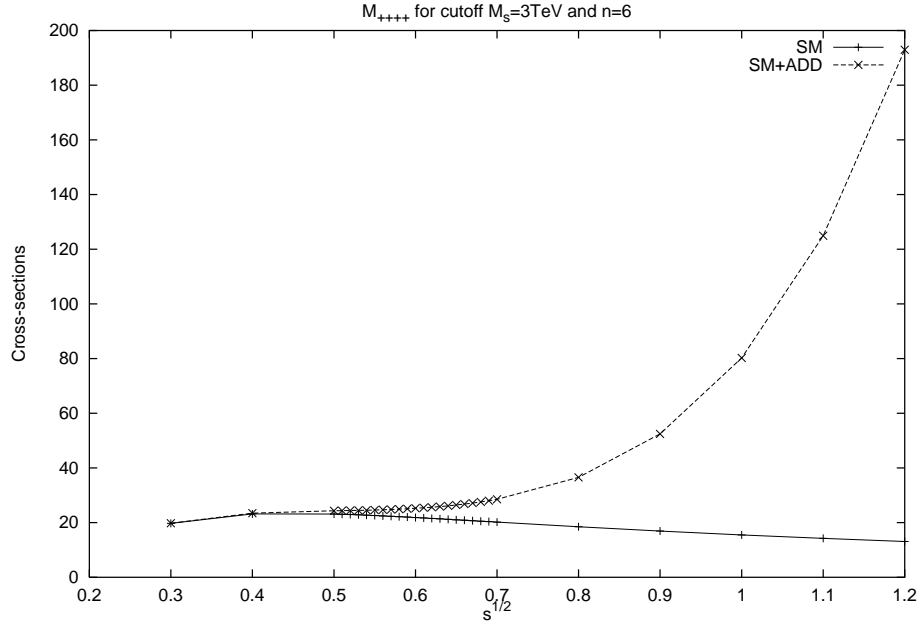


Figure 3: Magnitudes of the SM cross-sections and SM plus ADD cross-sections for initial helicities  $++$  for a cutoff of  $M_s = 3TeV$  and  $n=6$ .

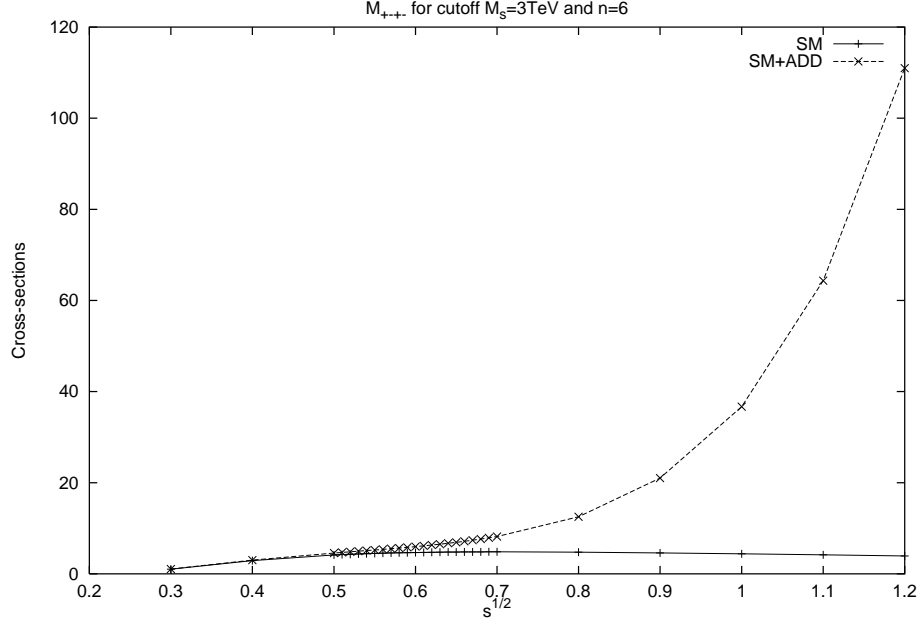


Figure 4: Magnitudes of the SM cross-sections and SM plus ADD cross-sections for initial helicities  $+-$  for a cutoff of  $M_s = 3TeV$  and  $n=6$ .

mentioning is the presence of resonance peaks only in the helicity state  $+-$  which should be a very clear signature should experiments be possible with polarized photons. The energy dependence of the cross-sections, even in the limited range considered is also very distinctive of the RS-scenario. The SM cross-sections decrease with energy whereas the total (SM+RS) cross-sections increase mildly with energy. Of course, these features would appropriately scaled as the mass of the first resonance takes on different values, but the qualitative nature of these features will continue. It should also be noted that these features are quite different from the cross-sections obtained by considering the ADD version WSQG as can be seen by comparison of figure 3 and 4 with their counterparts, figures 1 and 2.

In conclusion, experimental results on  $\gamma\gamma$  scattering in TeV scale would not only provide clear indications of possible departure from SM results not merely as a correction factor but by a large change in the value of cross-sections. Not only that, the values determined and their behaviour as a function of energy will enable us to distinguish between the two currently available scenarios of weak scale quantum gravity.

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